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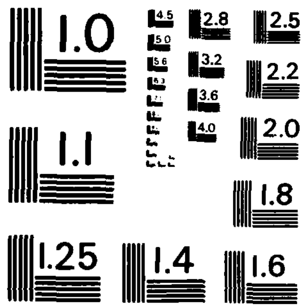

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AN INVESTIGATION OF SPIN-ORBIT RESONANCE

ABOUT A SEMI-SYNCHRONOUS NEAR POLAR ORBIT

THESIS

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THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

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Master of Science

by

Robert Ian Boren, B.S.

Capt USAF

Graduate Space Operations

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Robert Ian Boren

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A,B	Geopotential coefficients
a	Semi-major axis
E	Sum of $\Omega - n_0 t - \lambda_{22}$
e	Eccentricity
F, F', F^*	Hamiltonian
F_0	Unperturbed Hamiltonian
F_1	J_2 Hamiltonian contribution
F_2	Secular J_2^2 and J_4 Hamiltonian contribution
G,H,L	Delaunay elements
g, ω	Argument of perigee
h, Ω	Right ascension of the ascending node
I	Inclination
J_2, J_4, J_{22}	Geopotential harmonic coefficients
ℓ,M	Mean anomaly
n	Orbital mean motion
n_e	Mean rotation rate of the earth
Q,R,S	Generalized momenta - second transformation
R_e	Mean equatorial radius of the earth
r	Geocentric radial distance
S_2	Generating function
s	Generalized coordinate - second transformation
t	Time
U_{22}	Geopotential perturbing function
X,Y,Z	Generalized momenta - first transformation
x,y,z	Generalized coordinates - first transformation
β	Geocentric latitude

μ	Geocentric gravitational parameter
λ	Geocentric longitude
λ_{22}	Longitude associated with J_{22} harmonic
χ^*	Modified mean anomaly at epoch

Abstract

An investigation of the spin-orbit resonance effects for a semi-synchronous near polar orbit was undertaken in order to determine whether consideration of all resonance terms associated with a particular commensurability ratio would result in the existence of multiple equilibrium points for the placement of satellites utilizing this type of orbit. The Hamiltonian of the geopotential in Delaunay elements using first and second order zonal harmonic terms ~~(J_2, J_2^2, and J_4)~~ was first transformed to a set of modified variables. The effects of the resonant disturbing function were then developed and the resultant Hamiltonian, valid for near polar inclinations and small eccentricities, was reduced to a single degree of freedom through a second transformation. Phase portraits of the system Hamiltonian were then generated and an analysis of the resonance structure and librational periods performed.

A single equilibrium point for each resonance term was discovered. Because of the physical meaning of the critical argument~~, s,~~ the stable equilibrium points of both the first and second resonance terms appear to be candidates for deployment of future satellite systems. ~~In addition,~~ the semi-analytic technique of phase portraits was shown to be a feasible approach to the investigation of resonance effects.

I Introduction

The advent of the Space Age in 1957 brought with it recognition of the inherent advantages of observation platforms in orbit around the earth. Of interest to meteorological, land resources, and navigation satellites is a near polar orbit, which permits coverage of all latitudes of the earth's surface on a regular basis. It has also been recognized since the early days of space flight that satellites are subject to significant perturbations due to the non-spherical nature of the earth's potential. In particular, satellites having motions commensurate with the rotation rate of the earth experience a phenomenon known as spin-orbit resonance due to the elliptical shape of the earth's equatorial section. Probably the best known and most studied example of this phenomena is the existence of two stable equilibrium points for satellites in geosynchronous equatorial orbits. However, resonance effects have also been observed for other commensurate orbits. Of particular interest for satellites in a polar inclination is a semi-synchronous or 12-hour orbit. As the missions mentioned above generally require the utmost in orbital stability for purposes of data accuracy, any stable resonance points associated with this general type of orbit would be of extreme interest for positioning future satellites.

Historical Background

A review of the literature reveals that an extensive amount of research has been done concerning the effect of the earth's shape on satellite orbits. In particular, the effect of the earth's oblateness, or the J_2 zonal harmonic, has been well established by several researchers. In a classic edition, the November 1959 issue of The

Astronomical Journal contained three different and independent treatments of the "main problem" of satellite theory.

An offshoot of these early studies was the problem of motion near the critical inclination (63.4°) which can be considered a type of resonant motion. This, too, received much attention and is now well understood.

Further investigations have attempted to determine the effects of the earth's longitude dependent harmonics on satellite motion. Because of its use for communications satellites, the geosynchronous orbit has proven to be of the greatest interest.

Blitzer (Ref 7,8), using a linearized theory, has discussed equilibrium solutions for a geosynchronous satellite in a circular, equatorial orbit under the influence of the principle longitude dependent term, J_{22}. Blitzer (Ref 4,5) later treated the effect, due to higher order tesseral harmonics, on a geosynchronous satellite of small eccentricity and inclination. He showed that, due to the dominance of the J_{22} term, only a slight displacement of the equilibrium positions occurred, although there was a significant change in the librational periods.

Musen and Bailie (Ref 18), in 1962, studied the motion of synchronous satellites incorporating only the J_{22} tesseral harmonic and the J_2 and J_4 zonal harmonics. Their analytic technique agrees with Blitzer for synchronous motion in the equatorial plane.

Allan (Ref 1), in 1965, discussed the motion of nearly circular but inclined synchronous orbits. In 1967 (Ref 2) he studied the effect of resonance in inclination for synchronous satellites in nearly circular orbits when the positions of the nodes repeat relative to the rotating

primary. Later that same year, he also studied the effect of resonance in eccentricity and inclination for a synchronous satellite in a nearly equatorial, eccentric orbit which occurs when the longitude of the line of apsides repeats relative to the primary (Ref 3).

In a book published in 1966, Kaula (Ref 16) developed expressions for the resonant disturbing function of the geopotential in terms of inclination and eccentricity functions and derived general expressions for the variation of orbital elements due to arbitrary zonal or tesseral harmonics.

In a series of articles over the years, Garfinkel (Ref 13) has developed a formulation known as The Ideal Resonance Problem which has found wide application in resonance theory by treating specific cases as perturbations of the same.

Far more limited is the literature dealing with 12-hour resonant orbits. A 1961 article by Cook (Ref 10) deals with resonance effects for commensurate orbits in general but is restricted to orbits of low eccentricity and inclination. Blitzer (Ref 6), in 1963, gives a cursory treatment of 12 and 36-hour orbits in addition to the synchronous case but again only for the circular, near equatorial case. Allan, in the previously mentioned studies of 1967, also considers the problem of commensurate orbits other than the 24-hour case by calculating some resonance strengths, librational periods, and optimum inclinations using inclination functions.

A 1972 article by Wagner (Ref 21) studies the longitude drift regimes of eccentric 12-hour orbits utilizing the dominant terms of the geopotential. Inclinations near the critical inclination are specifically examined because of their possible long term stability.

Ten years later, Sochilina (Ref 20) investigated specific cases of 12-hour orbits at the critical inclination. In particular, he looks at orbits of the "Molniya" or "Navstar" type.

From the preceding, it is apparent that, while considerable progress has been made in resonance theory, it appears at this time that little hope exists for a general analytic solution except for certain specific cases. These include mostly satellites whose mean motion is strictly commensurate with the earth's rotation rate and whose orbits are at the critical inclination or have zero eccentricity. It should be noted that while Dallas and Diehl (Ref 11) claimed to have found such a general solution in 1977, they were later shown by Jupp (Ref 15) to have made a serious error. Garfinkel (Ref 13), in his 1979 summary of the Ideal Resonance Problem, lists the synchronous satellite with non-zero eccentricity and the general p:q resonance between the period of revolution of the satellite and the rotation of the earth as two of the outstanding unsolved problems of resonance theory.

As a result, although the effort continues to find formal, global solutions to resonance problems, most recent efforts address themselves to specific satellite orbits such as the studies by Wagner or Sochilina or attack the problem via a semi-analytic or numerical technique such as used by Nacozy and Diehl (Ref 19) in a recent article.

Of this latter category is the method of computer plotting phase portraits of the specific system Hamiltonian. Although this has been done previously in order to provide some sort of "physical feel," it would appear that little has been done to utilize the process as the primary investigative tool. While this technique provides information only in the regime of the particular inclination studied, it is valid

for small eccentricities, permitting investigation of all resonance terms associated with a particular commensurability ratio and is easily modified to incorporate additional harmonic terms or study different commensurabilities. But most importantly, it circumvents most of the mathematically sophisticated and algebraically laborious techniques required in the search for more general analytic solutions.

Objective

In an effort to better understand the regime of the semi-synchronous polar orbit, this study investigates the resonance effects due to the combination of certain zonal and tesseral harmonics of the geopotential via computer generation and analysis of phase portraits of the system Hamiltonian. It was suspected that consideration of all resonance terms associated with a particular harmonic term and commensurability ratio as a result of non-zero eccentricity would result in multiple equilibrium points. And, although not truly semi-synchronous, these points could sufficiently reduce station-keeping requirements so as to merit careful consideration for the deployment of future satellite systems.

Scope

Although the technique is easily modified to incorporate additional tesseral harmonics or commensurability ratios, the scope of this investigation is limited to the resonance effects arising from the interaction of the first and second order zonal effects of the geopotential, i.e., the J_2, J_2^2, and J_4 terms, with the principal longitude dependent term, J_{22}, on a semi-synchronous, near polar orbit.

General Approach

The method of investigation revolves around utilizing a two-body Hamiltonian of the system that already contains the zonal harmonic terms. The Hamiltonian, which is expressed in terms of Delaunay variables, is then converted by a canonical transformation to a set of variables that will prove more convenient in the development of the disturbing function. The appropriate resonance terms of the J_{22} tesseral harmonic are then selected and transformed, assuming small eccentricity, for inclusion in the system Hamiltonian. Next, a second canonical transformation is used to eliminate variables and reduce to a single degree of freedom. The computer is then used to plot phase portraits of the Hamiltonian of each resonance term. These show the number and structure of the *stable equilibrium points*. In addition, the program determines the width of the stable region, the libration period about the stable point, and its location relative to a nominal semi-synchronous orbit.

Sequence of Presentation

Chapter II presents the theoretical development of the system Hamiltonian and equations of motion for each resonance term. Chapter III discusses the computer analysis, Chapter IV presents the results, and Chapter V provides an interpretation and a brief discussion of the results as well as recommendations for further study.

II Theoretical Development

Phase Portraits

Prior to actually developing the equations necessary for this investigation, some justification is in order by way of a brief discussion of the theory of phase portraits.

In Hamiltonian mechanics, the motion of a dynamical system with n degrees of freedom can be represented geometrically by the motion of a representative point P in a 2n-space defined by the n generalized coordinates q_k and the n conjugate momenta p_k. This 2n space, called the phase space, can be considered a 2n dimensional Euclidean space. The physical difference between q_k and p_k poses no problem since they possess equal status in Hamiltonian mechanics.

In the phase space, the point P traces a path or trajectory that corresponds to a particular solution of the dynamical problem. For different initial conditions, the motion will be described by a different trajectory. The real advantage of the geometric representation becomes evident when we consider not just one trajectory but the totality of trajectories, known as the phase portrait, representing all possible solutions.

Of course, the representation of the motion of a system with n degrees of freedom in a 2n dimensional phase space is itself difficult and hard to visualize. If the system could be reduced to a single degree of freedom, the motion could be portrayed on a two dimensional phase plane which is easier to comprehend. Hence, if constants of the motion could be obtained by elimination of variables, a complex dynamical system could be easily portrayed on a two dimensional phase portrait. This can be done relatively easily by performing a canonical

transformation of variables.

The goal of much of the rest of this chapter is to develop the Hamiltonian for a semi-synchronous near polar orbit and by a canonical transformation, reduce it to a single degree of freedom so that the resonance effects can be portrayed on a two dimensional phase portrait.

Zonal Hamiltonian

In order to study the resonance effects for any type of orbit, the Hamiltonian for a two-body system that already contains the effects of the included zonal harmonics must be obtained. Ignoring the long period terms, Hori (Ref 14:292), after the method of Brouwer (Ref 9), gives the Hamiltonian through second order as

$$F = F_0 + F_1 + F_2 \tag{1}$$

with

$$F_0 = \frac{\mu^2}{2L^2} \tag{2}$$

$$F_1 = \frac{\mu^4 J_2 R_e^2}{2L^3G^3}\left(-\frac{1}{2} + \frac{3}{2}\left(\frac{H}{G}\right)^2\right) \tag{3}$$

$$\begin{aligned} F_2 = \frac{\mu^6 J_2^2 R_e^4}{4L^5G^5}\Big\{&\frac{15}{32} + \frac{27}{32}\frac{J_4}{J_2^2} - \left(\frac{27}{16} + \frac{135}{16}\frac{J_4}{J_2^2}\right)\left(\frac{H}{G}\right)^2 \\ &+ \left(\frac{15}{32} + \frac{315}{32}\frac{J_4}{J_2^2}\right)\left(\frac{H}{G}\right)^4 + \frac{3}{8}\frac{L}{G}\left(1 - 6\left(\frac{H}{G}\right)^2 + 9\left(\frac{H}{G}\right)^4\right) \\ &- \left(\frac{L}{G}\right)^2\left[\frac{15}{32} + \frac{45}{32}\frac{J_4}{J_2^2} - \left(\frac{15}{16} + \frac{225}{16}\frac{J_4}{J_2^2}\right)\left(\frac{H}{G}\right)^2 - \left(\frac{105}{32} - \frac{525}{32}\frac{J_4}{J_2^2}\right)\left(\frac{H}{G}\right)^4\right]\Big\} \end{aligned} \tag{4}$$

where μ is the gravitational parameter of the earth, R_e its mean equatorial radius, and L, G, and H are the momenta terms of the Delaunay action-angle variables. These are related to the standard Keplerian elements by

$L = \sqrt{\mu a}$, $\ell = M$ = mean anomaly,

$G = L\sqrt{1 - e^2}$, $g = \omega$ = argument of perigee,

$H = G \cos I$, $h = \Omega$ = right ascension of ascending node,

where a and e are the osculating semi-major axis and eccentricity, respectively, and I is the instantaneous inclination. The subscripts of F represent the order of the associated zonal harmonic, J_2 being first order and J_2^2 and J_4 of the second order.

First Transformation

It will prove convenient in the development of the disturbing function to have an eccentricity related variable that is itself small for small eccentricities. This can be done by performing a canonical transformation to a new set of variables. Defining the new momenta variables as

$$X = L = \sqrt{\mu a}$$

$$Y = L - G = \sqrt{\mu a}[1 - \sqrt{1 - e^2}]$$

$$Z = H = (X - Y) \cos I$$

the generating function becomes

$$S_2 = X\ell + (X - Y)g + Zh \tag{5}$$

and

$$x = \ell + g, \qquad y = - g, \qquad z = h$$

become the new angle variables. Converting to canonical units ($R_e = \mu = 1$), the zonal Hamiltonian now becomes

$$F' = F'(X,Y,X,x,y,z,t) = F(L,G,H,\ell,g,h,t) = F_0 + F_1 + F_2 \tag{6}$$

where

$$F_0 = \frac{1}{2X^2} \tag{7}$$

$$F_1 = \frac{J_2}{2X^3(X-Y)^3}\left(-\frac{1}{2} + \frac{3}{2}\left(\frac{Z}{X-Y}\right)^2\right) \tag{8}$$

$$\begin{aligned} F_2 = \frac{J_2^2}{4X^5(X-Y)^5}\Big\{&\frac{15}{32} + \frac{27}{32}\frac{J_4}{J_2^2} - \left(\frac{27}{16} + \frac{135}{16}\frac{J_4}{J_2^2}\right)\left(\frac{Z}{X-Y}\right)^2 \\ &+ \left(\frac{15}{32} + \frac{315}{32}\frac{J_4}{J_2^2}\right)\left(\frac{Z}{X-Y}\right)^4 + \frac{3}{8}\frac{X}{X-Y}\left(1 - 6\left(\frac{Z}{X-Y}\right)^2 + 9\left(\frac{Z}{X-Y}\right)^4\right) \\ &- \left(\frac{X}{X-Y}\right)^2\Big[\frac{15}{32} + \frac{45}{32}\frac{J_4}{J_2^2} - \left(\frac{15}{16} + \frac{225}{16}\frac{J_4}{J_2^2}\right)\left(\frac{Z}{X-Y}\right)^2 \\ &- \left(\frac{105}{32} - \frac{525}{32}\frac{J_4}{J_2^2}\right)\left(\frac{Z}{X-Y}\right)^4\Big]\Big\} \end{aligned} \tag{9}$$

Disturbing Function

At this point, the resonance disturbing function can be developed and incorporated into the Hamiltonian.

The disturbance at a point due to the principle tesseral harmonic term of the geopotential, J_{22}, is normally given as

$$U_{22} = \frac{\mu}{r}\left(\frac{R_e}{r}\right)^2 J_{22} P_2^2(\sin\beta)\cos 2(\lambda - \lambda_{22}) \tag{10}$$

where r, β, and λ are the geocentric radius, latitude and longitude of the point, λ_{22} is the longitude of the major axis of the earth's equator, and $P_2^2(\sin\beta)$ is the associated Legendre function. Inclusion in the Hamiltonian usually entails expanding Eq (10) in powers of the eccentricity. At this point, however, a slight short-cut is in order. From Allan (Ref 1:62), the general form of the disturbing function in canonical units (R_e = μ 1) can also be expressed as

$$U_{\ell m} = \frac{J_{\ell m}}{a^{\ell+1}}\sum_{p,q}^{\ell} F_{\ell mp}(I)G_{\ell pq}(e)\cos[(\ell-2p)\omega + (\ell-2p+q)M + m(\Omega-n_e t-\lambda_{\ell m})] \tag{11}$$

where a, e, I, Ω, ω, and M are the Keplerian elements, n_e is the mean rotation rate of the earth such that $n_e t$ is the Greenwich sidereal time,

and $F_{\ell mp}(I)$ and $G_{\ell pq}(e)$ are inclination and eccentricity functions for which formulas or existing tables are available (Ref 16).

To pick out the resonant terms, note that the rate of change of the argument of Eq (11) is

$$(\ell-2p)\dot{\omega} + (\ell-2p+q)(n+\dot{\chi}^*) + m(\dot{\Omega}-n_0)$$

where χ^* is the modified mean anomaly at epoch defined by

$$M = \int n dt + \chi^*$$

and n is the orbital mean motion. Since $\dot{\omega}$, $\dot{\chi}^*$, and $\dot{\Omega}$ are small for a semi-synchronous polar orbit, this is approximately equal to

$$(\ell-2p+q)n - mn_0$$

For resonance to occur, this must be equal to zero and since $n \cong 2n_0$ for a semi-synchronous orbit, the resonance condition is given when $m = 2(\ell-2p+q)$. Discarding the non-resonant terms leaves the following general expression for the resonant terms:

$$U_{\ell m} = \frac{J_{\ell m}}{a^{\ell+1}} \sum_{p}^{\ell} F_{\ell mp}(I)G_{\ell pq}(e)\cos[(\ell-2p)\omega + (\ell-2p+q)M + mE] \tag{12}$$

where $E = \Omega-n_0t-\lambda_{\ell m}$ and $q = m/2-\ell+2p$. Therefore, for the J_{22} term ($\ell = m = 2$) p = 0,1,2 and q = -1,1,3 which gives F_{220}, F_{221}, and F_{222} as the inclination functions and G_{20-1}, G_{211}, and G_{223} as the associated eccentricity functions.

From Kaula (Ref 16:34-38) the inclination functions are

$$F_{220} = \frac{3}{4}(1 + \cos I)^2 , \qquad F_{221} = \frac{3}{2}\sin^2 I , \qquad F_{222} = \frac{3}{4}(1 - \cos I)^2$$

and the eccentricity functions to $O(e^7)$ are

$$G_{20-1} = (-\frac{e}{2} + \frac{e^3}{16} - \frac{5e^5}{384} - \frac{143e^7}{18432})$$

$$G_{211} = \left(\frac{3e}{2} + \frac{27e^3}{16} + \frac{261e^5}{128} + \frac{143097e^7}{6144}\right)$$

$$G_{223} = \left(\frac{e^3}{48} + \frac{11e^5}{768} + \frac{313e^7}{30720}\right)$$

which gives for the resonant disturbing function for J_{22}

$$U_{22} = \frac{3J_{22}}{4a^3}(1 + \cos I)^2\left(-\frac{e}{2} + \frac{e^3}{16} - \frac{5e^5}{384} - \frac{143e^7}{18432}\right)\cos(M + 2E + 2\omega)$$
$$+ \frac{3J_{22}}{2a^3}\sin^2 I\left(\frac{3e}{2} + \frac{27e^3}{16} + \frac{261e^5}{128} + \frac{143097e^7}{6144}\right)\cos(M + 2E)$$
$$+ \frac{3J_{22}}{4a^3}(1 - \cos I)^2\left(\frac{e^3}{48} + \frac{11e^5}{768} + \frac{313e^7}{30720}\right)\cos(M + 2E - 2\omega) \quad (13)$$

It is worth noting at this point, that resonance effects on a semi-synchronous orbit only exist for non-circular cases since there are no zero order terms in the eccentricity functions.

Before adding to the zonal Hamiltonian, the disturbing function must now be converted to the modified variables. Since

$$Z = (X - Y)\cos I \quad (14)$$

then

$$\cos I = \frac{Z}{X-Y} \quad (15)$$

and letting

$$B = \frac{Z}{X-Y} = \cos I \quad (16)$$

gives

$$(1 + \cos I)^2 = (1 + B)^2 \quad (17)$$

$$\sin^2 I = 1 - B^2 \quad (18)$$

$$(1 - \cos I)^2 = (1 - B)^2 \quad (19)$$

and if

$$Y = X(1 - \sqrt{1 - e^2}) \tag{20}$$

then

$$e = \sqrt{[2Y/X - (Y/X)^2]} \tag{21}$$

Now, because of the earlier change of variables, if e is constrained to be small means that Y is also small and, as a result, the higher orders of Y/X can be discarded in Eq (21) as well as the higher orders of e in the resonant disturbing function.

Making the appropriate substitution of variables into Eq (13) gives

$$\begin{aligned} U_{22} = &\frac{3J_{22}}{4X^6}(1 + B)^2(-\frac{1}{2}\sqrt{2Y/X})\cos(x - y + 2z - 2n_e t - 2\lambda_{22}) \\ &+ \frac{3J_{22}}{2X^6}(1 - B^2)(\frac{3}{2}\sqrt{2Y/X})\cos(x + y + 2z - 2n_e t - 2\lambda_{22}) \\ &+ \frac{3J_{22}}{4X^6}(1 - B)^2(\frac{Y}{24X}\sqrt{2Y/X})\cos(x + 3y + 2z - 2n_e t - 2\lambda_{22}) \end{aligned} \tag{22}$$

Simplifying results in

$$\begin{aligned} U_{22} = &-\frac{3J_{22}\sqrt{Y}}{4\sqrt{2X^{13}}}(1 + B)^2\cos(x - y + 2z - 2n_e t - 2\lambda_{22}) \\ &+ \frac{9J_{22}\sqrt{Y}}{2\sqrt{2X^{13}}}(1 - B^2)\cos(x + y + 2z - 2n_e t - 2\lambda_{22}) \\ &+ \frac{3J_{22}\sqrt{Y^3}}{48\sqrt{2X^{15}}}(1 - B)^2\cos(x + 3y + 2z - 2n_e t - 2\lambda_{22}) \end{aligned} \tag{23}$$

<u>Second Transformation</u>

To investigate the individual resonance terms, each one is incorporated separately with the zonal Hamiltonian. This is possible because for the polar orbit the effects of the individual terms are distinctly different. In the immediate vicinity of each resonance, the effects of that term dominate the others and may be treated

independently. In order to do this, the explicit time dependence must be removed. This can be accomplished by means of a second canonical transformation. At the same time this process can be used to reduce the Hamiltonian to a single degree of freedom by eliminating variables. This is done by grouping most of the angle variables in the cosine function together into a single critical argument.

In order to make the transformation perfectly general for all three resonance terms as well as those of other commensurability ratios let

$$s = ix + jy + kz - mn_0 t$$

where i, j, k, and m are integers,

$$q = x$$

and

$$r = z$$

The generating function, S_2, then becomes

$$S_2 = Qx + S(ix + jy + kz - mn_0 t) + Rz \tag{24}$$

yielding

$$X = Q + iS, \qquad Y = jS, \qquad Z = kS + R$$

as the relationship between the old and new momenta. The new Hamiltonian then becomes

$$F^* = F^*(Q,R,S,\_,\_,s,\_) = F'(X,Y,Z,x,y,z,t) - \frac{\partial S_2}{\partial t}$$

$$= F_0 + F_1 + F_2 + U_{22} + mn_0 S \tag{25}$$

with

$$F_0 = \frac{1}{2(Q+iS)^2} \tag{26}$$

$$F_1 = \frac{J_2}{2(Q+iS)^6} A^3(-\frac{1}{2} + \frac{3}{2}B^2) \tag{27}$$

$$
\begin{aligned}
F_2 = \frac{J_2^2}{4(Q+iS)^{10}} A^5\{ & \frac{15}{32} + \frac{27}{32}\frac{J_4}{J_2^2} - (\frac{27}{16} + \frac{135}{16}\frac{J_4}{J_2^2})B^2 \\
& + (\frac{15}{32} + \frac{315}{32}\frac{J_4}{J_2^2})B^4 + \frac{3}{8}A(1 - 6B^2 + 9B^4) \\
& - A^2[\frac{15}{32} + \frac{45}{32}\frac{J_4}{J_2^2} - (\frac{15}{16} + \frac{225}{16}\frac{J_4}{J^2})B^2 \\
& - (\frac{105}{32} - \frac{525}{32}\frac{J_4}{J_2^2})B^4]\}
\end{aligned}
\qquad (28)
$$

where

$$A = \frac{Q+iS}{Q+(i-j)S} \qquad B = \frac{kS+R}{Q+(i-j)S}$$

and U_{22} is the corresponding resonance term. In the current case, only the index j changes. As a result, the only changes for each resonant Hamiltonian are the actual resonance term and the coefficients A and B.

Now, since q and r do not appear in the new Hamiltonian, Q and R are constants of the motion. In addition, since time does not appear in the Hamiltonian, the total energy of the system is a constant also and is equal to the Hamiltonian. Hence, the system has been reduced to a single degree of freedom where phase portraits of the coordinate s and the conjugate momentum S for various values of the Hamiltonian can be plotted by the computer. To do this, however, requires determining the location of the resonance equilibrium points. Since the equilibrium points are relative extremas of the system Hamiltonian, i.e., an energy well or peak, their location is determined by where the derivatives of the Hamiltonian are equal to zero. As the Hamiltonian is a function of both S and s, the derivative with respect to both of them will be required. These derivatives will also prove necessary in the actual computer plotting since they define the slope of the Hamiltonian at a given point.

Derivatives

Note that since s only appears as an argument of the cosine,

$$\frac{\partial F^*}{\partial s} = - U_{22} \tag{29}$$

with the cosine replaced by the sine. The other partial is not quite as straightforward. In order to save some effort, $\partial F^*/\partial S$ for the zonal part can also be done in a general form. Liberal application of the chain rule to Eqs (26)-(28) and a lot of "attempted simplification" results in the following expressions for the partials of the zonal terms

$$\frac{\partial F_0}{\partial S} = - \frac{i}{(Q+iS)^3} \tag{30}$$

$$\frac{\partial F_1}{\partial S} = - \frac{3J_2}{2(Q+iS)^7} A^3\{[\frac{(2i-j)Q+(2i^2-2ij)S}{Q+(i-j)S}](-\frac{1}{2} + \frac{3}{2}B^2)$$

$$- AB[\frac{kQ-(i-j)R}{Q+(i-j)S}]\} \tag{31}$$

$$\frac{\partial F_2}{\partial S} = - \frac{5J_2^2}{4(Q+iS)^{11}} A^5[\{\frac{(2i-j)Q+(2i^2-2ij)S}{Q+(i-j)S}\}\{\frac{15}{32} + \frac{27}{32}\frac{J_4}{J_2^2} - (\frac{27}{16} + \frac{135}{16}\frac{J_4}{J_2^2})B^2$$

$$+ (\frac{15}{32} + \frac{315}{32}\frac{J_4}{J_2^2})B^4 + \frac{3}{8}A(1 - 6B^2 + 9B^4) - A^2[\frac{15}{32} + \frac{45}{32}\frac{J_4}{J_2^2}$$

$$- (\frac{15}{16} + \frac{225}{16}\frac{J_4}{J_2^2})B^2 - (\frac{105}{32} - \frac{525}{32}\frac{J_4}{J_2^2})B^4]\}$$

$$- \frac{A}{5}\{[\frac{kQ-(i-j)R}{Q+(i-j)S}]\langle - (\frac{27}{8} + \frac{135}{8}\frac{J_4}{J_2^2})B + (\frac{15}{8} + \frac{315}{8}\frac{J_4}{J_2^2})B^3$$

$$+ \frac{3}{8}A(-12B + 36B^3) + A^2[(\frac{15}{8} + \frac{225}{8}\frac{J_4}{J_2^2})B + (\frac{105}{8} - \frac{525}{8}\frac{J_4}{J_2^2})B^3]\rangle$$

$$+ [\frac{jQ}{Q+(i-j)S}]\langle\frac{3}{8}(1 - 6B^2 + 9B^4) - 2A[\frac{15}{32} + \frac{45}{32}\frac{J_4}{J_2^2} - (\frac{15}{16} + \frac{225}{16}\frac{J_4}{J_2^2})B^2$$

$$- (\frac{105}{32} - \frac{525}{32}\frac{J_4}{J_2^2})B^4]\rangle\}] \tag{32}$$

The specific Hamiltonian and derivatives for investigation of each individual resonance term can now be determined.

First Resonance Term

Taking the argument of the first term of Eq (21) gives transformation indices of i = 1, j = -1, k = 2, and m = 2. Transforming the resonant term and applying the indices to Eq (27) results in the following resonant Hamiltonian

$$
\begin{aligned}
F^* = {} & \frac{1}{2(Q+S)^2} + \frac{J_2}{2(Q+S)^6} A^3\left(-\frac{1}{2} + \frac{3}{2}B^2\right) \\
& + \frac{J_2^2}{4(Q+S)^{10}} A^5\left\{\frac{15}{32} + \frac{27}{32}\frac{J_4}{J_2^2} - \left(\frac{27}{16} + \frac{135}{16}\frac{J_4}{J_2^2}\right)B^2\right. \\
& \qquad + \left(\frac{15}{32} + \frac{315}{32}\frac{J_4}{J_2^2}\right)B^4 + \frac{3}{8}A(1 - 6B^2 + 9B^4) \\
& \qquad - A^2\left[\frac{15}{32} + \frac{45}{32}\frac{J_4}{J_2^2} - \left(\frac{15}{16} + \frac{225}{16}\frac{J_4}{J_2^2}\right)B^2\right. \\
& \qquad \left.\left. - \left(\frac{105}{32} - \frac{525}{32}\frac{J_4}{J_2^2}\right)B^4\right]\right\} \\
& - \frac{3J_{22}\sqrt{-S}}{4\sqrt{2(Q+S)^{15}}}(1 + B)^2\cos(s - 2\lambda_{22}) + 2n_0 S
\end{aligned}
\tag{33}
$$

where

$$A = \frac{Q+S}{Q+2S} \qquad \text{and} \qquad B = \frac{2S+R}{Q+2S}$$

Likewise, taking the partial of the resonant term and applying the indices to Eq (29) gives

$$
\begin{aligned}
\frac{\partial F^*}{\partial S} = {} & -\frac{1}{(Q+S)^3} - \frac{3J_2}{2(Q+S)^7} A^3\left\{\left[\frac{3Q+4S}{Q+2S}\right]\left(-\frac{1}{2} + \frac{3}{2}B^2\right) - AB\left[\frac{2Q-2R}{Q+2S}\right]\right\} \\
& - \frac{5J_2^2}{4(Q+S)^{11}} A^5\left[\left(\frac{3Q+4S}{Q+2S}\right)\left\{\frac{15}{32} + \frac{27}{32}\frac{J_4}{J_2^2} - \left(\frac{27}{16} + \frac{135}{16}\frac{J_4}{J_2^2}\right)B^2\right.\right. \\
& + \left(\frac{15}{32} + \frac{315}{32}\frac{J_4}{J_2^2}\right)B^4 + \frac{3}{8}A(1 - 6B^2 + 9B^4) - A^2\left[\frac{15}{32} + \frac{45}{32}\frac{J_4}{J_2^2}\right. \\
& \left.\left.\left. - \left(\frac{15}{16} + \frac{225}{16}\frac{J_4}{J_2^2}\right)B^2 - \left(\frac{105}{32} - \frac{525}{32}\frac{J_4}{J_2^2}\right)B^4\right]\right\}\right.
\end{aligned}
$$

$$- \frac{A}{5}\{[\frac{2Q-2R}{Q+2S}]<-(\frac{27}{8} + \frac{135}{8}\frac{J_4}{J_2^2})B + (\frac{15}{8} + \frac{315}{8}\frac{J_4}{J_2^2})B^3$$

$$+ \frac{3}{8}A(-12B + 36B^3) + A^2[(\frac{15}{8} + \frac{225}{8}\frac{J_4}{J_2^2})B + (\frac{105}{8} - \frac{525}{8}\frac{J_4}{J_2^2})B^3]>$$

$$- [\frac{Q}{Q+2S}]<\frac{3}{8}(1 - 6B^2 + 9B^4) - 2A[\frac{15}{32} + \frac{45}{32}\frac{J_4}{J_2^2} - (\frac{15}{16} + \frac{225}{16}\frac{J_4}{J_2^2})B^2$$

$$- (\frac{105}{32} - \frac{525}{32}\frac{J_4}{J_2^2})B^4]>\}]$$

$$- \frac{3J_{22}}{8\sqrt{-2S(Q+S)^{15}}}[(12S-Q)(1 + B)^2 - (\frac{2Q-2R}{Q+2S})4SA(1 + B)]\cos(s - 2\lambda_{22})$$

$$+ 2n_0 \tag{34}$$

and

$$\frac{\partial F^*}{\partial s} = \frac{3J_{22}\sqrt{-S}}{4\sqrt{2(Q+S)^{13}}}(1 + B)^2 \sin(s - 2\lambda_{22}) \tag{35}$$

Note that the appearance of $\sqrt{-S}$ is taken care of by the fact that the specific form of the transformation results in S itself being negative. This has some implications for the computer implementation.

Second Resonance Term

Taking the second term of Eq (21) gives indices of $i = 1$, $j = 1$, $k = 2$, and $m = 2$. The only part of the Hamiltonian that changes is the resonant term

$$U_{22} = \frac{9J_{22}\sqrt{S}}{2\sqrt{2(Q+S)^{11}}}(1 - B^2)\cos(s - 2\lambda_{22}) \tag{36}$$

and the coefficients

$$A = \frac{Q+S}{Q} \quad \text{and} \quad B = \frac{2S+R}{Q}$$

The partials of F^* are now

$$\frac{\partial F^*}{\partial S} = - \frac{1}{(Q+S)^3} - \frac{3J_2}{2(Q+S)^7} A^3\{(-\frac{1}{2} + \frac{3}{2}B^2) - 2AB\}$$

$$- \frac{5J_2^2}{4(Q+S)^{11}} A^5[(\frac{15}{32} + \frac{27}{32}\frac{J_4}{J_2^2} - (\frac{27}{16} + \frac{135}{16}\frac{J_4}{J_2^2})B^2$$

$$+ (\frac{15}{32} + \frac{315}{32}\frac{J_4}{J_2^2})B^4 + \frac{3}{8}A(1 - 6B^2 + 9B^4) - A^2[\frac{15}{32} + \frac{45}{32}\frac{J_4}{J_2^2}$$

$$- (\frac{15}{16} + \frac{225}{16}\frac{J_4}{J_2^2})B^2 - (\frac{105}{32} - \frac{525}{32}\frac{J_4}{J_2^2})B^4])$$

$$- \frac{A}{5}\{2<- (\frac{27}{8} + \frac{135}{8}\frac{J_4}{J_2^2})B + (\frac{15}{8} + \frac{315}{8}\frac{J_4}{J_2^2})B^3$$

$$+ \frac{3}{8}A(- 12B + 36B^3) + A^2[(\frac{15}{8} + \frac{225}{8}\frac{J_4}{J_2^2})B + (\frac{105}{8} - \frac{525}{8}\frac{J_4}{J_2^2})B^3]>$$

$$+ <\frac{3}{8}(1 - 6B^2 + 9B^4) - 2A[\frac{15}{32} + \frac{45}{32}\frac{J_4}{J_2^2} - (\frac{15}{16} + \frac{225}{16}\frac{J_4}{J_2^2})B^2$$

$$- (\frac{105}{32} - \frac{525}{32}\frac{J_4}{J_2^2})B^4]>\}]$$

$$- \frac{9J_{22}}{4\sqrt{2S(Q+S)^{15}}}[(12S-Q)(1 - B^2) + 8SAB]\cos(s - 2\lambda_{22}) + 2n_0 \qquad (37)$$

and

$$\frac{\partial F^*}{\partial s} = - \frac{9J_{22}\sqrt{S}}{2\sqrt{2(Q+S)^{13}}}(1 - B^2)\sin(s - 2\lambda_{22}) \qquad (38)$$

Third Resonance Term

The third term of Eq (21) gives indices of $i = 1$, $j = 3$, $k = 2$, and $m = 2$. Again, the only change in F^* is the resonant term

$$U_{22} = \frac{9J_{22}\sqrt{3S^3}}{48\sqrt{2(Q+S)^{15}}}(1 - B)^2\cos(s - 2\lambda_{22}) \qquad (39)$$

and the coefficients

$$A = \frac{Q+S}{Q-2S} \qquad \text{and} \qquad B = \frac{2S+R}{Q-2S}$$

The partials of F^* become

$$\frac{\partial F^*}{\partial S} = - \frac{1}{(Q+S)^3} - \frac{3J_2}{2(Q+S)^7}A^3\{[\frac{-Q-4S}{Q-2S}](- \frac{1}{2} + \frac{3}{2}B^2) - AB[\frac{2Q+2R}{Q-2S}]\}$$

$$
\begin{aligned}
&- \frac{5J_2^2}{4(Q+S)^{11}} A^5 [\{(\frac{-Q-4S}{Q-2S})\{\frac{15}{32} + \frac{27}{32}\frac{J_4}{J_2^2} - (\frac{27}{16} + \frac{135}{16}\frac{J_4}{J_2^2})B^2 \\
&+ (\frac{15}{32} + \frac{315}{32}\frac{J_4}{J_2^2})B^4 + \frac{3}{8}A(1 - 6B^2 + 9B^4) - A^2[\frac{15}{32} + \frac{45}{32}\frac{J_4}{J_2^2} \\
&- (\frac{15}{16} + \frac{225}{16}\frac{J_4}{J_2^2})B^2 - (\frac{105}{32} - \frac{525}{32}\frac{J_4}{J_2^2})B^4]\} \\
&- \frac{A}{5}\{[\frac{2Q+2R}{Q-2S}]\langle - (\frac{27}{8} + \frac{135}{8}\frac{J_4}{J_2^2})B + (\frac{15}{8} + \frac{315}{8}\frac{J_4}{J_2^2})B^3 \\
&+ \frac{3}{8}A(-12B + 36B^3) + A^2[(\frac{15}{8} + \frac{225}{8}\frac{J_4}{J_2^2})B + (\frac{105}{8} - \frac{525}{8}\frac{J_4}{J_2^2})B^3]\rangle \\
&+ [\frac{3Q}{Q-2S}]\langle\frac{3}{8}(1 - 6B^2 + 9B^4) - 2A[\frac{15}{32} + \frac{45}{32}\frac{J_4}{J_2^2} - (\frac{15}{16} + \frac{225}{16}\frac{J_4}{J_2^2})B^2 \\
&- (\frac{105}{32} - \frac{525}{32}\frac{J_4}{J_2^2})B^4]\rangle\}] \\
&- \frac{3J_{22}\sqrt{3S}}{32\sqrt{2(Q+S)^{17}}}[(12S-3Q)(1 - B)^2 + (\frac{2Q+2R}{Q-2S})4SA(1 - B)]\cos(s - 2\lambda_{22}) \\
&+ 2n_e \qquad (40)
\end{aligned}
$$

and

$$
\frac{\partial F^*}{\partial s} = - \frac{9J_{22}\sqrt{3S^3}}{48\sqrt{2(Q+S)^{15}}}(1 - B)^2 \sin(s - 2\lambda_{22}) \qquad (41)
$$

Equilibrium Conditions

Location of the stable and unstable equilibrium points is determined by where the two partial derivatives of F^* are both equal to zero. By inspection of Eqs (35), (38), and (41), one can see that all three resonances will have equilibrium points where

$$
s = 2\lambda_{22}
$$

and

$$
s = \pi + 2\lambda_{22}
$$

Determination of the values of S and the exact structure of the resonances, however, will require computer analysis as explained in

Chapter III. Before that can be done, though, one last development needs to be performed.

Libration Period

Since an object not placed right at a stable equilibrium tends to librate or oscillate in semi-major axis and longitude about the point, it would be useful to know the libration period for each resonance region. Since only the region around the stable point is of major interest, only the period for small displacements near the stable point need be calculated. This is normally done analytically by use of elliptic integrals. Again, a short cut is available.

In the neighborhood of an equilibrium point of a function such as a stable point which is a minima of the Hamiltonian, the equations of motion may be linearized about the equilibrium point (Ref 17:180) such that

$$\delta\dot{S} = \frac{\partial\dot{S}}{\partial s}\delta s \tag{42}$$

and

$$\delta\dot{s} = \frac{\partial\dot{s}}{\partial S}\delta S \tag{43}$$

for some small displacement δs or δS about the stable point. But, from Hamilton's equations

$$\dot{S} = \frac{\partial F^*}{\partial s} \tag{44}$$

and

$$\dot{s} = -\frac{\partial F^*}{\partial S} \tag{45}$$

Substituting Eqs (44) and (45) into Eqs (42) and (43) yields

$$\delta\dot{S} = \frac{\partial^2 F^*}{\partial s^2}\delta s \tag{46}$$

and

$$\delta\dot{s} = -\frac{\partial^2 F^*}{\partial S^2}\delta S \tag{47}$$

This is a system of coupled constant coefficient differential equations. Since the second partials are constant when evaluated at the equilibrium point, taking the time derivative of Eq (47) gives

$$\delta\ddot{s} = -\frac{\partial^2 F^*}{\partial S^2}\delta\dot{S} \tag{48}$$

Substituting Eq (46) into Eq (48) for $\delta\dot{S}$ results in

$$\delta\ddot{s} = -\frac{\partial^2 F^*}{\partial S^2}\frac{\partial^2 F^*}{\partial s^2}\delta s \tag{49}$$

This is immediately recognized as the standard form of a harmonic oscillator whose rotational frequency is

$$\omega = \left[\left(\frac{\partial^2 F^*}{\partial S^2}\right)\left(\frac{\partial^2 F^*}{\partial s^2}\right)\right]^{\frac{1}{2}} \tag{50}$$

the period of which is

$$T = \frac{2\pi}{\omega} \tag{51}$$

Hence, by numerically evaluating the second partials for a small displacement near the stable point where the first partial is known to be zero, the librational period for small oscillations about the stable point can be determined.

III Computer Analysis

With the necessary equations now developed, the next step was to implement the computer analysis of the effects of the three different resonance terms. Because of the extremely complex nature of the equations, this analysis was performed on a large CDC mainframe computer.

Software

The appendix contains a commented listing of the routines (set up for the first term) used in the analysis. They were implemented in FORTRAN V because of the availability of a contouring routine in that language and the author's familiarity with FORTRAN. The four main routines used or developed, are briefly described in the following sections.

Program RESPLT. RESPLT is the driver program for the software. It is responsible for run initialization, program control, and phase portrait plotting. The program itself can actually be broken down into four main functions. The first is to initialize all the program constants including those for the geopotential model. Next, the plot is initialized by drawing and labeling of the plot axes. Third, the stable and unstable equilibrium points as well as resonance width and libration period are determined by a call to RESN. And, finally, the phase portrait is plotted by making successive calls to CONTUR to obtain the data points.

The actual plotting was done on a CALCOMP plotter and was facilitated by the availability of an extensive library of plot routines on the CDC computer.

Subroutine RESN. RESN is the routine responsible for calculation of many of the parameters associated with the resonance term of interest. It begins by determining the nominal two-body radius for a semi-synchronous orbit. Then, using a simple binary search algorithm, it searches out the location of the unstable and stable equilibrium points based on the equations developed in Chapter II. Next, again using a binary search algorithm, it locates the edges of the stable resonance band and determines the resonance width. It does this by taking the value of the Hamiltonian at the unstable point and, since the energy contours passing through the unstable point define the edges of the stable region, it searches along the radius through the stable point for the same values. Finally, it determines the libration period for small oscillations about the stable point.

Subroutine CONTUR. The CONTUR subroutine is the real heart of the phase portrait plotting. The routine was provided by Dr. William Wiesel of the Astronautics Department. Since the variables to be plotted on the phase portrait, the conjugate momentum S and coordinate s, are action-angle variables, the routine did have to be modified slightly to permit contouring of a function in polar coordinates instead of Cartesian.

The routine itself uses a two variable Newton iteration scheme to locate successive values of S and s for a constant energy contour of the Hamiltonian given initial values of S and s on the contour. The derivatives or slopes of the Hamiltonian are used to determine the direction the routine needs to iterate in order to follow the contour.

Subroutine FDF. The routine FDF implements the equations developed in Chapter II for each resonance term of interest. Given values of the

variables S and s as well as the constants, it evaluates the value of the coefficients A and B, the Hamiltonian, and its derivatives for use by the calling routine.

Procedure

Investigation of the three resonance terms actually began with the software debugging process which continued throughout the study due to quirks peculiar to each term. After numerous computer runs, though, a fair amount of confidence in the approach was developed.

One of the first decisions to be made was the choice of the geopotential model. Since differences among models are generally minor, the 1973 Smithsonian Standard Earth (Ref 12) was chosen because of its ready availability. Table I gives a listing of the requisite constants based on this model.

Table I

SAO III Geopotential Constants

Constant	Symbol	Value
Earth Radius	R_e	6378140 meters
Canonical Time Unit	TU	806.8108 sec
Earth Rotation Rate	n_e	0.0588335721 rad/TU
First Order Zonal Term	J_2	1082.637×10^{-6}
Second Order Zonal Term	J_4	-1.617999×10^{-6}
Principal Tesseral Term	J_{22}	2.7438636×10^{-6}
Tesseral Longitude	λ_{22}	$-14.923723°$

A related question was the values to be assigned to the constants of motion Q and R. Since the second transformation relates Q, R and S to the orbital inclination, a value of zero was chosen for R so as to

result in consideration of inclinations very near but not at 90° for small values of S. The exact value of the inclination at an equilibrium point then depends on Q and the resulting value of S at the point. The choice of Q was equally as arbitrary. Since Q and S are also related to the eccentricity, values of Q for each resonance were chosen that would result in a value of S at the stable point that corresponded to an eccentricity of about 0.01. This was considered to be both a realistic and realizable eccentricity for satellites in this type of orbit. This proved to be an iterative process because of the impact of the resonance structure itself and the resultant plotting considerations.

It was also decided at this time to set the J_{22} longitude term, λ_{22}, to zero. This was done primarily as a plotting convenience. Since this term only serves to rotate the orientation of the phase portrait in terms of the angle s, its inclusion adds little insight to the problem. It could just as well have been incorporated in the critical argument of the second transformation.

The next step was to run a simplified version of RESN by itself in order to determine the approximate location of the resonance structures prior to actually attempting to plot them.

With these preliminary steps taken care of, full scale investigation of the resonance terms proceeded. The actual process consisted of making "several" full scale computer runs on each resonance term in order to select a Q and, hence, the resulting values of S for each term. The outputs of these runs were some of the resonance parameters associated with each term as well as polar contour plots of the conjugate action-angle variables S and s for various values of the Hamiltonian.

This process required a certain amount of manual interaction because of the individual resonance quirks mentioned previously. For example: the first resonance required plotting -S because of the second transformation. The second resonance was fairly routine but the third required major massaging because of its extremely fine structure.

The specific results stemming from this process are detailed in the next chapter.

IV Results

The specific results of this investigation are presented in the following pages. They are grouped by resonance term as defined by the equations of Chapter II. Tables II, III, and IV list the parameters associated with each resonance term and Figures 1, 2, and 3 are the corresponding phase portraits.

Note that the values of Q and S are expressed in canonical units where $Q+S = \sqrt{\mu a}$, a in terms of earth radii (ER). Equilibrium point radius is also given in ER. This is equivalent to the semi-major axis at the equilibrium point. The distance of the equilibrium points from nominal is referenced to a nominal two-body semi-synchronous radius of 4.1645031 ER.

The phase portraits are standard polar contour plots of the action (momentum) variable S (or -S) against the angle variable s for equal values of the system Hamiltonian, F^*. Examination of the plots indicate the existence of one stable and one unstable equilibrium point (marked by Xs) associated with each term. The stable point is distinguished by the closed contours around it.

An interpretation and complete discussion of these results and the resonance effects is presented in the next chapter.

Table II

First Resonance Term

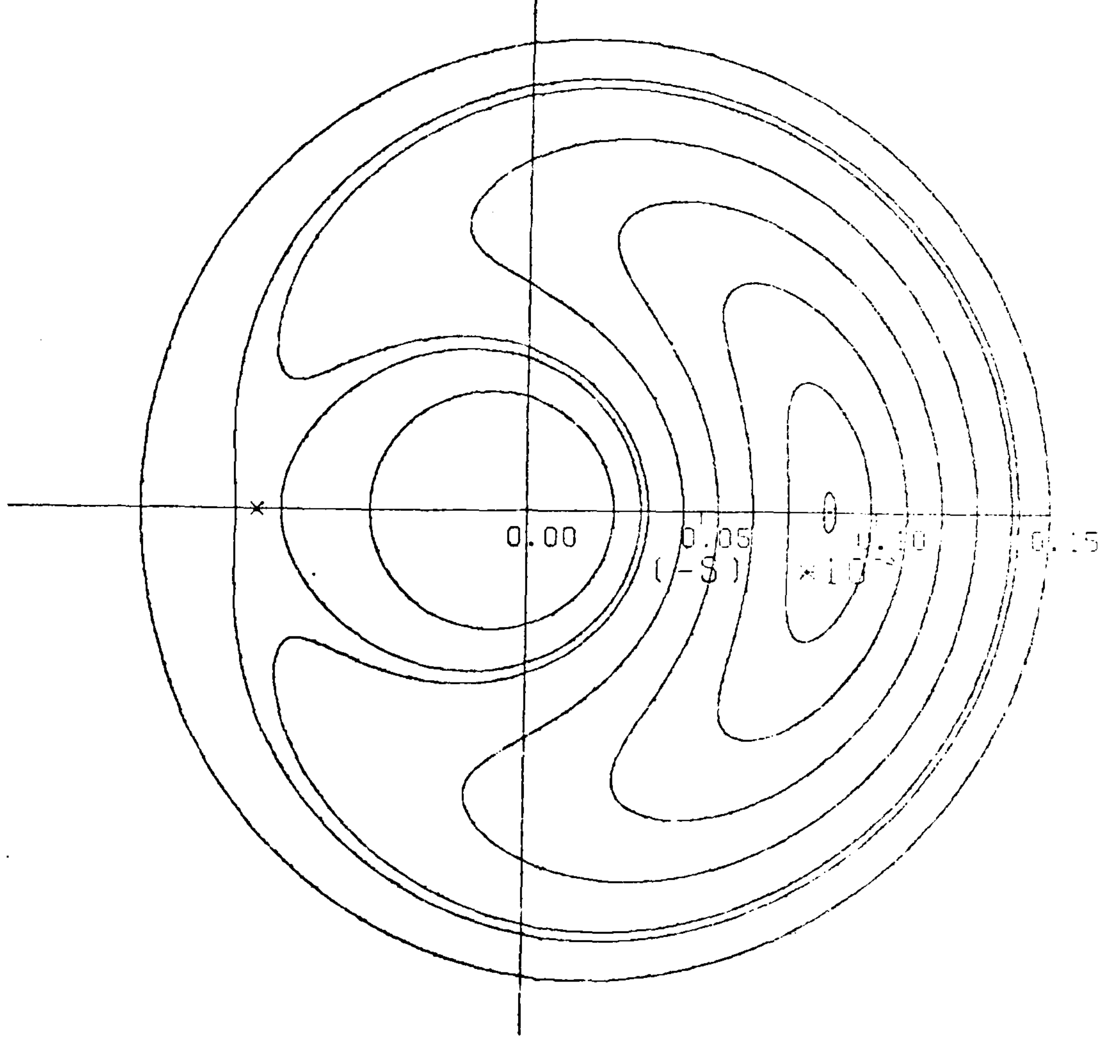
($Q = 2.04079386$)

Unstable Equilibrium Point

S =	$-$ 0.00007779 ER^2/TU
s =	180.0°
Radius =	4.16452206 ER
Nominal Displacement =	120.8145 meters

Stable Equilibrium Point

S =	$-$ 0.00008679 ER^2/TU
s =	0.0°
Radius =	4.16448533 ER
Nominal Displacement =	$-$ 113.4052 meters
Orbit Inclination (I) =	90.005°
Orbit Eccentricity (e) =	0.009
Resonance Width =	2791.9897 meters
Libration Period =	12,188.8 days



1ST RESONANCE TERM

(Q=2.04079386)

Figure 1. First Resonance Term

Table III

Second Resonance Term

(Q = 2.04048689)

Unstable Equilibrium Point

S =	0.00020756 ER^2/TU
s =	0.0°
Radius =	4.16443385 ER
Nominal Displacement =	- 441.7858 meters

Stable Equilibrium Point

S =	0.00024027 ER^2/TU
s =	180.0°
Radius =	4.16456734 ER
Nominal Displacement =	409.6754 meters
Orbit Inclination (I) =	89.987°
Orbit Eccentricity (e) =	0.015
Resonance Width =	8740.8379 meters
Libration Period =	3,843.1 days

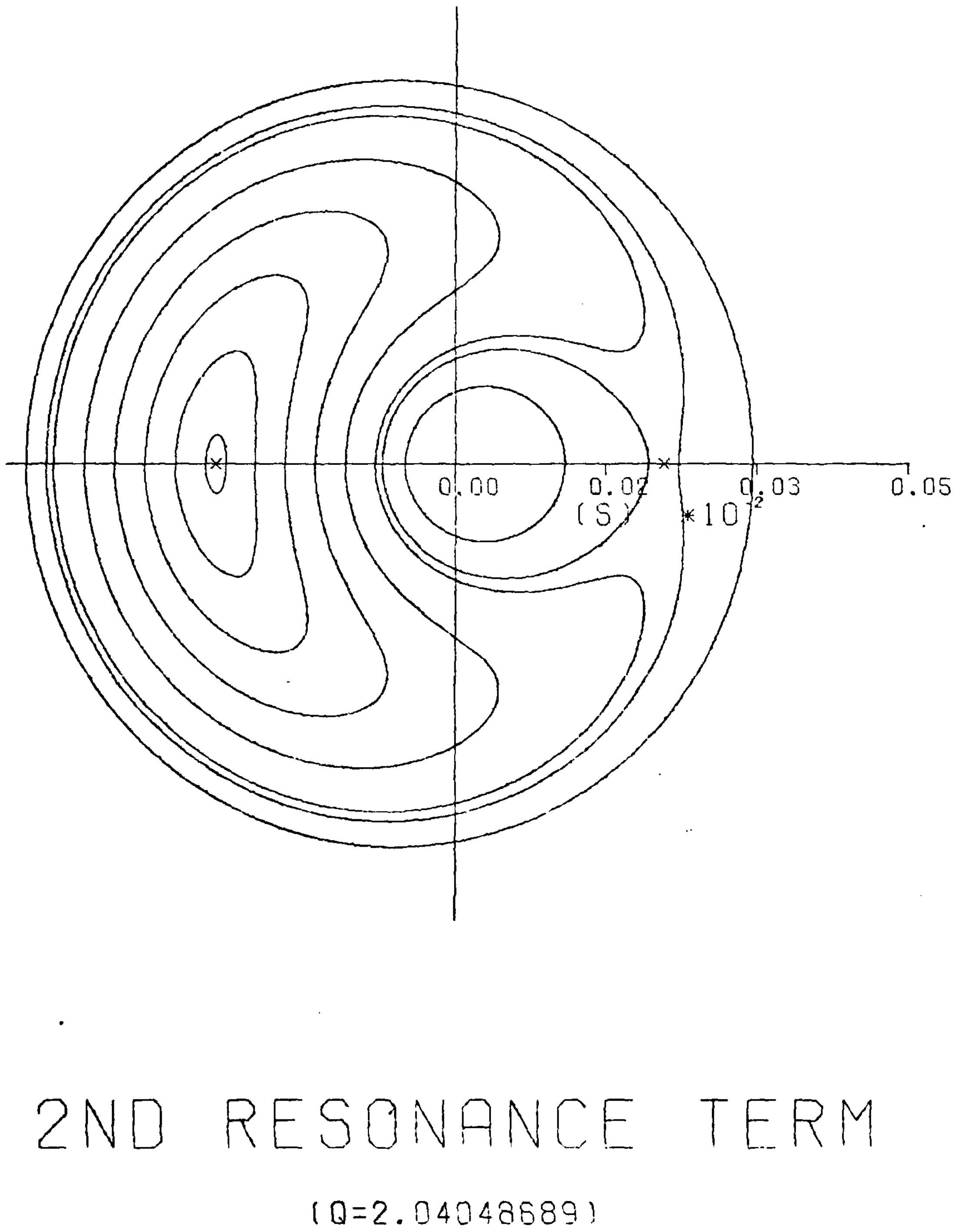


Figure 2. Second Resonance Term

Table IV

Third Resonance Term

(Q = 2.04068145)

Unstable Equilibrium Point

S =	0.0002996 ER^2/TU
s =	0.0°
Radius =	4.16450307 ER
Nominal Displacement =	- 0.2783 meters

Stable Equilibrium Point

S =	0.00002996 ER^2/TU
s =	180.0°
Radius =	4.16450307 ER
Nominal Displacement =	- 0.2708 meters
Orbit Inclination (I) =	89.998°
Orbit Eccentricity (e) =	0.009
Resonance Width =	5.546 meters
Libration Period =	6,385,139.3 days

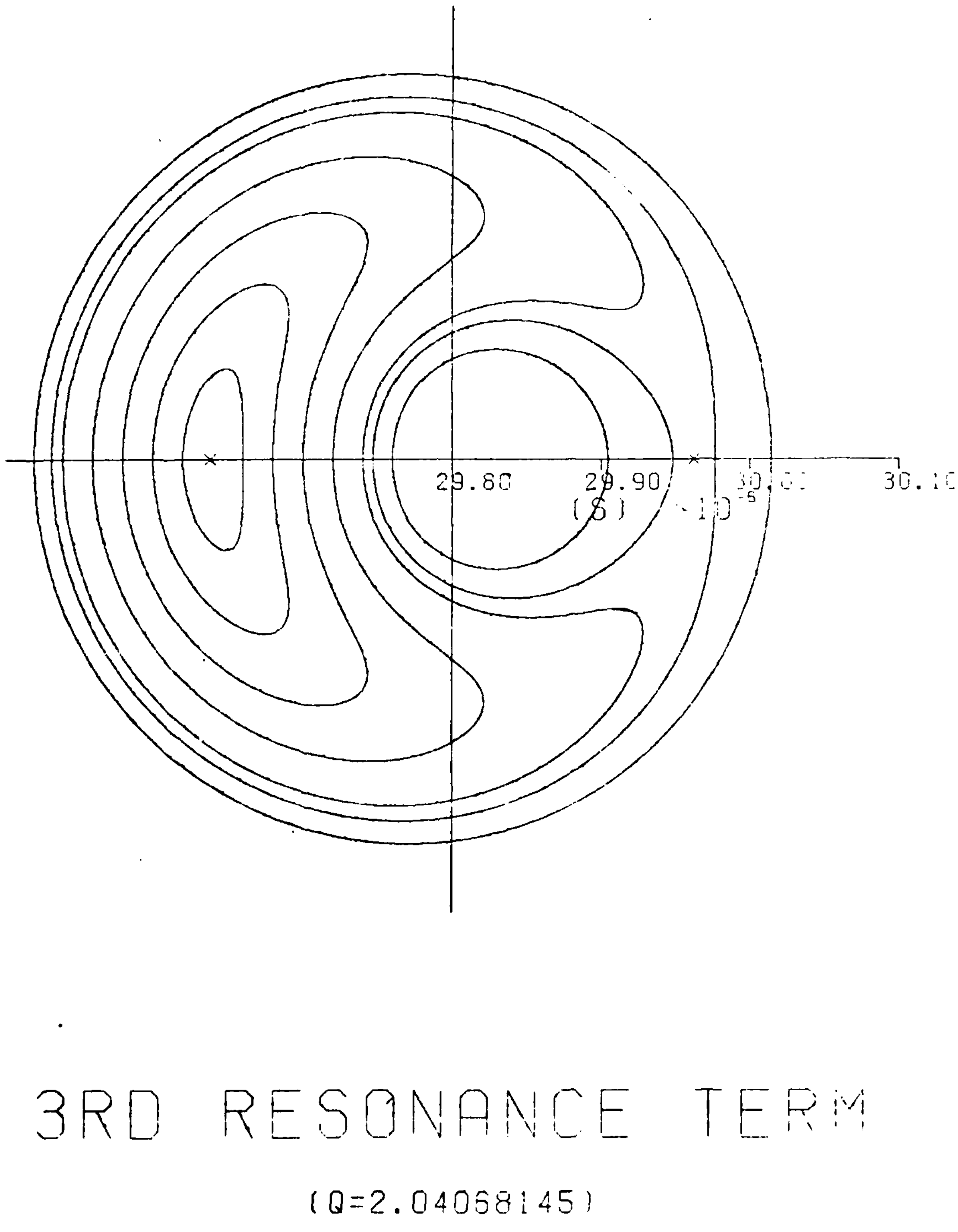


Figure 3. Third Resonance Term

V Discussion and Recommendations

Discussion

One of the goals of this study was to determine if consideration of all resonance terms associated with a particular tesseral harmonic and commensurability ratio would result in multiple usable equilibrium points. From a first glance, the results do not look particularly promising. Investigation of the structure of all three resonance terms indicate the existence of only a single stable point associated with each for non-circular orbits. In addition, two other aspects of the stable points raise questions about their suitability.

First, note that all three stable points are displaced slightly from nominal semi-synchronous altitude. The broader and stronger the resonance, the greater the displacement. This results in objects placed at each stable point having mean motions different from each other as well as from nominal semi-synchronous causing potential phase drifts between the different terms. The actual physical implications of this will be made clear later.

Secondly, the stable region due to the third term is so shallow and narrow that it may prove impossible to use. With the resonance being only 5.5 meters wide, it would be difficult to position a satellite in that small a region at that altitude to begin with. Even if one were placed there, the resonance is so weak, being of $O(e^3)$, that it would require only a minor external perturbation to push it out, thereby negating its value.

Although the third term does not appear a likely candidate for use, a closer look at the first and second terms may be merited. This is because that, while the position of a stable point in terms of the

momentum variable S is easily interpreted as determining the semi-major axis of the resonant orbit, the physical meaning of the angle variable s is not as clear-cut. Because of the second transformation, s actually incorporates several different non-parallel orbital angles, each of which may possess secular variations. This makes it difficult to visualize just what the resonant condition is for a polar orbit without closer study.

<u>First Resonance Term</u>. Returning to Eq (13) to express s in terms of the Delaunay elements gives

$$\ell_1 + 2h_1 - 2n_e t + 2g_1 - 2\lambda_{22} = 0$$

for the stable point of the first resonance. Initially setting $g_1 = 0$ and solving for the longitude of the node by setting $\ell_1 = 0$ gives

$$h_1 - n_e t = \lambda_{22}$$

where $h - n_e t$ is the geocentric longitude of the ascending node. In the same manner, solving for the longitude of the descending node by setting $\ell_1 = \pi$ gives

$$h_1 - n_e t = \lambda_{22} - \frac{\pi}{2}$$

In other words, resonant stability for this case is determined not only by the semi-major axis of the orbit but by also locating the ascending node of the orbit over the major axis of the earth's equatorial section and the descending node over the minor axis. This is certainly intuitively satisfying from a physical point of view. A similar analysis performed on the unstable point where

$$\ell_1 + 2h_1 - 2n_e t + 2g_1 - 2\lambda_{22} = \pi$$

gives

$$h_1 - n_0 t = \lambda_{22} + \frac{\pi}{2}$$

for the longitude of the ascending node and

$$h_1 - n_0 t = \lambda_{22}$$

for the descending node. This indicates that unstable resonance results from the first term when the ascending node of the orbit is over the minor axis and the descending node is over the major axis. Unlike the geosynchronous equatorial case where resonance is a function of semi-major axis and satellite longitude, in the polar case it is a function of semi-major axis and node location.

The preceding analysis was based on an argument of perigee of zero. Now consider what happens if some other argument of perigee is selected. If $g_1 = \pi/4$ then

$$\ell_1 + 2h_1 - 2n_0 t + \frac{\pi}{2} - 2\lambda_{22} = 0$$

for the stable point. This yields

$$h_1 - n_0 t = \lambda_{22} - \frac{\pi}{4}$$

for the longitude of the ascending node and

$$h_1 - n_0 t = \lambda_{22} - \frac{3\pi}{4}$$

for the longitude of the descending node. Again, this is understandable from a physical point of view. Since a rotation of the perigee causes the time from ascending node to descending node to differ from the time from descending to ascending, a change in the node is necessary to balance the effects of the earth's equatorial section and maintain the resonance condition. In general, a rotation of the perigee location

results in an equivalent regression of the line of nodes such that, for $g_1 = \pi/2$

$$h_1 - n_0 t = \lambda_{22} - \frac{\pi}{2}$$

for the longitude of the ascending node and

$$h_1 - n_0 t = \lambda_{22} - \pi$$

for the descending node so that resonant stability is now determined by an ascending node over the longitude of the minor axis and descending node over the major axis. A further rotation of perigee to π results in ascending and descending nodes being on the opposite ends of the major and minor axes then for the $g_1 = 0$ case. A similar analysis on the unstable equilibrium point shows the same effect.

This opens up an array of opportunities for deploying satellite systems. Simply by careful selection of the longitude of ascending node and the argument of perigee for each orbit, a constellation of almost any size, depending on mission requirements, can be established that utilizes the first term resonance effects. Orbits of common nodal longitude can be used to provide the same ground tracks while different arguments of perigee will determine the particular node and, hence, its ground track. In addition, since they all depend on the first term, they all have the same semi-major axis and, as a result, the same mean motion, resulting in a constant phasing among the satellites of the constellation. The last issue raises one final consideration. The previous analysis has shown that resonance for a semi-synchronous near polar orbit is fundamentally a locking of the nodes of the orbit in phase with the axes of the earth's equatorial section. It was pointed out earlier, however, that the altitudes of the stable points are

displaced slightly from nominal semi-synchronous altitude, slightly below in the case of the first resonance. An object placed at the stable semi-major axis of the first resonance will have a mean motion of 2.00001281 revolutions/day. This will result in a difference between the longitude of the nodes and the major and minor axes of 0.0023°/day. As a result, one effect of the resonance is to cause a secular variation in the right ascension of -0.0023°/day in order to maintain the phasing of the nodes with the axes of the earth's equator. In truth, because of the interaction of h and g for the first term, this variation is imposed on both as a drift in both h and g, the exact components of which are more difficult to separate. This effect, however, provides an explanation of why the stronger the resonance and, hence, the greater the secular variation, the greater the displacement from nominal altitude.

Second Resonance Term. Taking the argument of the second term from Eq (13) gives

$$\ell_2 + 2h_2 - 2n_0 t - 2\lambda_{22} = \pi$$

for the stable point. Setting $\ell_2 = 0$ then yields

$$h_2 - n_0 t = \lambda_{22} + \frac{\pi}{2}$$

as the longitude of the ascending node. In the same manner, letting $\ell_2 = \pi$ results in

$$h_2 - n_0 t = \lambda_{22}$$

for the longitude of the descending node. Therefore, for the second resonance term, resonant stability is established by placing ascending node over the minor axis and descending node over the major. Performing the same analysis on the unstable point where

$$\ell_2 + 2h_2 - 2n_0t - 2\lambda_{22} = 0$$

gives

$$h_2 - n_0t = \lambda_{22}$$

for ascending node and

$$h_2 - n_0t = \lambda_{22} - \frac{\pi}{2}$$

for descending node or ascending at major axis and descending at minor. For the second term, however, since the argument of perigee does not enter into the argument, rotation of the perigee does not alter the resonant condition. As a result, stable resonance is always characterized by the fixed relationship of the nodes to the equatorial axes. This makes the second term somewhat less interesting than the first.

Now, the stable altitude of the second term also imposes a secular variation on the line of nodes. Its semi-major axis is equivalent to a mean motion of 1.99995373 revolutions/day. As a result, the right ascension must precess 0.0083°/day in order to maintain the resonant condition. In this case, the effect is totally in right ascension since argument of perigee does not appear in the argument.

Third Resonance Term. For completeness, the third term will be briefly addressed. Again, taking the argument from Eq (13) for the stable point gives

$$\ell_3 + 2h_3 - 2n_0t - 2g_3 - 2\lambda_{22} = \pi$$

Setting $g_3 = 0$ results in

$$h_3 - n_0t = \lambda_{22} + \frac{\pi}{2}$$

for the longitude of ascending node and

$$h_3 - n_0 t = \lambda_{22}$$

for descending node. For the unstable point where

$$\ell_3 + 2h_3 - 2n_0 t - 2g_3 - 2\lambda_{22} = 0$$

gives

$$h_3 - n_0 t = \lambda_{22}$$

for ascending node and

$$h_3 - n_0 t = \lambda_{22} - \frac{\pi}{2}$$

for descending node. As a result, the basic resonant condition is identical to that of the second term, however, since the argument of perigee appears in the argument, a rotation of perigee results in a rotation of the line of nodes similar to that of the first term but in the opposite direction. The stable altitude of the third point gives a mean motion of 2.00000003 revolutions/day which results in a resonant variation in right ascension of 0.0000056°/day in order to stay in phase.

Because of its closer proximity to a nominal semi-synchronous altitude, the third term might be the most interesting were it not for its relative weakness. In fact, because of the widths of the first and second resonances which would encompass the third for a common resonant longitude, an object in the third would more likely be dominated by the stronger term and tend to librate within that resonance. The same result could occur for the first term, since the 8700 meter width of the second term totally contains the first at a common stable longitude.

Libration Periods. This last thought raises the point of libration again. From the numerical results presented in the last chapter, it can be seen that all three resonance terms have relatively long libration

periods making the orbital changes due to libration small over short periods of time. Because of the interpretation just given for the resonance variables, the libration is actually seen as an oscillation in the semi-major axis and longitude of ascending node of the orbit for a satellite not placed exactly at a stable point. The further the displacement in either direction from the stable point, the longer the libration period until it falls outside the stable region where it exhibits a secular motion in longitude known as circulation.

Although not a lot of data is available on semi-synchronous libration periods, a couple of studies have obtained analytical values for other inclinations that provide some comparison. Allan (Ref 2:63) obtained a value of approximately 2265 days for a semi-synchronous orbit with an inclination of 34.4°. Sochilina (Ref 20:345-348) gives approximately 5000 days as the librational period at the critical inclination. These appear to compare favorably with the value of 3851 days for the dominant second term obtained numerically in this study.

Recommendations

From the preceding, it is obvious that there does exist a definite opportunity for exploiting resonance effects on a semi-synchronous near polar orbit. As was shown in the discussion, the effect of the argument of perigee in the first resonance term provides significant flexibility in the deployment of satellites utilizing resonance effects. This is something that has been overlooked in the past because of the tendency to consider only circular orbits, thus eliminating any resonance effects at all, or to treat only the dominant term which in this case does not contain the argument of perigee effect. As a result, this study has shown that consideration of all resonance terms does provide additional

stable positions which because of their physical interpretation do not interfere with each other as much as might be expected. These results certainly merit consideration in the deployment of future satellite systems. In this regard, there are two additional areas worth looking at.

One would be to study the added effects of additional tesseral harmonics. Even though they are substantially smaller that the J_{22} term, they may affect the results enough to require consideration.

Another area of investigation that this study had hoped to address was to determine the station keeping requirements in the vicinity of the stable points. This could prove to be the real test of the feasibility of using these regions for the deployment of future satellite systems.

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Appendix

Program and Subroutine Listings

The following is a sample listing of the program and three main subroutines (set up for the first resonance) used in the process of this investigation. The routines called for plotting are from a general purpose plotter subroutine library for the CDC.

```
      PROGRAM RESPLT
C  *****************
C     DRIVER PROGRAM FOR INVESTIGATION AND PLOTTING
C     OF RESONANCE EFFECTS DUE TO EARTH'S GEOPOTENTIAL
C  *****************
      DIMENSION BIGS(9000), SMLS(9000)
      REAL J2,J2SQ,J4,J4OVR,J22,NO
      COMMON /SAOIII/ J2,J2SQ,J4,J4OVR,J22,NO,PHI22
      COMMON /PARAM/ Q,R,RADIAN,TPI
C     SET GEOPOTENTIAL AND HAMILTONIAN CONSTANTS
      J2 = 1082.637E-6
      J2SQ = J2**2
      J4 = -1.617999E-6
      J4OVR = J4/J2SQ
      J22 = 2.7438636E-6
      NO = 5.86729371E-2
      PHI22 = 0.0
      Q = 2.04265452
      R = 0.0
      PI = 4.0*ATAN(1.0)
      TPI = 2.0 * PI
      RADIAN = 360./TPI
C     SCALE AND INITIALIZE PLOT
      SCALE = .00005
      CALL PLOTS(0,0,1)
      CALL PLOT(0.0,-.5,-3)
      CALL PLOT(0.0,0.0,-3)
      CALL PLOT(1.0,7.0,3)
      CALL PLOT(4.0,7.0,2)
      CALL AXIS(4.0,7.0,'(-S)',-4,3.,0.,0.0,SCALE)
      CALL PLOT(4.0,4.0,3)
      CALL PLOT(4.0,10.0,2)
      CALL PLOT(1.2,2.5,3)
      CALL SYMBOL(1.23,2.5,0.315,'1ST RESONANCE TERM',0.,18)
      CALL SYMBOL(3.05,2.0,0.14,'(Q=2.04265452)',0.,14)
      CALL PLOT(0.0,0.0,3)
      SI = -.000025
```

```
C     CALCULATE AND PLOT EQUILIBRIUM POINTS
      CALL RESN(SI,STBLE,UNSTB)
      XS = 4.0 + ABS(STBLE)/SCALE
      XU = 4.0 - ABS(UNSTB)/SCALE
      CALL SYMBOL(XS,7.0,0.07,4,0.,-1)
      CALL SYMBOL(XU,7.0,0.07,4,0.,-1)
C ****************
C     LOOP TO PLOT CONTOURS
C ****************
      DO 4 I = 1,10
C     INITIALIZE FOR INDIVIDUAL CONTOUR
      BIGS(1) = -SI
      BSNEW = SI
      SMLS(1) = 0.0
      SMLSNW = 0.0
      IFLG = 0
      N = 1
      ICAS = 0
      DEL = TPI/3600.
C     DETERMINE POINTS ON EACH CONTOUR
    1 CALL CONTUR(BSNEW,SMLSNW,DEL,BSNEW,SMLSNW,IOK,ICAS,FS)
      IF (IOK.EQ.0) GO TO 2
      PRINT *,'BGSO=',BIGS(N),' BIGS=',BSNEW,' SMLS=',SMLSNW,' FSTR=',FS
      GO TO 3
    2 N = N + 1
      BIGS(N) = -BSNEW
      SMLS(N) = SMLSNW * RADIAN
      IF (N.EQ.8998) GO TO 3
C     END OF CONTOUR?
      IF ((SMLSNW.GE.0.0).AND.(IFLG.EQ.1)) GO TO 3
      IF (SMLSNW.LT.0.0) IFLG = 1
      IF (SMLSNW.LT.TPI) GO TO 1
    3 BIGS(N+1) = 0.0
      BIGS(N+2) = SCALE
C ****************
C     PLOT EACH CONTOUR
C ****************
      CALL POLAR(BIGS,SMLS,-N,4.0,7.0,3.0,0,3)
C     SET BIG S FOR START OF NEXT CONTOUR
      SI = SI - .00001
      IF (I.EQ.7) SI = -.000033
      IF (I.EQ.8) SI = -.000141
      IF (I.EQ.9) SI = -.00015
    4 CONTINUE
      K = 0
C     TERMINATE PLOT
      CALL PLOTE(K)
      STOP
      END
```

```
      SUBROUTINE RESN(SI,STBLE,UNSTB)
C  *****************
C     ROUTINE TO CALCULATE RESONANCE PARAMETERS
C     SI IS INITIAL BIG S TO START SEARCH
C  *****************
      REAL J2,J2SQ,J4,J4OVR,J22,NO
      COMMON /SAOIII/ J2,J2SQ,J4,J4OVR,J22,NO,PHI22
      COMMON /PARAM/ Q,R,RADIAN,TPI
C     INITIALIZE CONSTANTS AND VARIABLES
      ER = 6378140.0
      BGS = SI
      DS = -7 * SI
      SMS = TPI/2.0
C     CALCULATE NOMINAL SEMI-SYNCHRONOUS RADIUS
      SR = (2*NO)**(-2.0/3.0)
      PRINT *,' '
      PRINT *,'NOMINAL RADIUS (DU)'
      PRINT *,SR
C  *****************
C     LOCATE UNSTABLE POINT
C  *****************
    1 DS = DS/2.0
      CALL FDF(BGS,SMS,FSTR,DFDBS,DFDSS)
      BGS = BGS - DS * SIGN(1.0,DFDBS)
      IF (DS.GT.1E-16) GO TO 1
      FUNSTB = FSTR
      UNSTB = BGS
      RAD = (Q + BGS)**2
      RADMET = (RAD - SR)*ER
      PRINT *,' '
      PRINT *,'UNSTABLE EQUILIBRIUM POINT'
      PRINT *,' '
      PRINT *,'S=',UNSTB,' RADIUS (DU)=',RAD,' DIST FM NOM (M)=',RADMET,
     2 ' FSTR=',FUNSTB
C     REINITIALIZE
      BGS = SI
      DS = -7 * SI
      SMS = 0.0
C  *****************
C     LOCATE STABLE POINT
C  *****************
    2 DS = DS/2.0
      CALL FDF(BGS,SMS,FSTR,DFDBS,DFDSS)
      BGS = BGS - DS * SIGN(1.0,DFDBS)
      IF (DS.GT.1E-16) GO TO 2
      STBLE = BGS
      RAD = (Q + BGS)**2
      RADMET = (RAD-SR)*ER
      PRINT *,' '
      PRINT *,'STABLE EQUILIBRIUM POINT'
      PRINT *,' '
      PRINT *,'S=',STBLE,' RADIUS (DU)=',RAD,' DIST FM NOM (M)=',RADMET,
     2 ' FSTR=',FSTR,' DFDBS=',DFDBS,' DFDSS=',DFDSS
```

```
C  *****************
C     CALCULATE RESONANCE WIDTH
C  *****************
      DS = 5 * SI
C     LOCATE INSIDE EDGE OF STABLE REGION
    3 DS = DS/2.0
      CALL FDF(BGS,SMS,FSTR,DFDBS,DFDSS)
      FDIF = FUNSTB - FSTR
      BGS = BGS - DS * SIGN(1.0,FDIF)
      IF (ABS(DS).GT.1E-16) GO TO 3
      EDGE1 = BGS
      PRINT *,' '
      PRINT *,'INSIDE EDGE=',EDGE1
      ED1R = (Q+BGS)**2
      BGS = STBLE
      DS = 5 * SI
C     LOCATE OUTSIDE EDGE OF STABLE REGION
    4 DS = DS/2.0
      CALL FDF(BGS,SMS,FSTR,DFDBS,DFDSS)
      FDIF = FUNSTB - FSTR
      BGS = BGS + DS * SIGN(1.0,FDIF)
      IF (ABS(DS).GT.1E-16) GO TO 4
      EDGE2 = BGS
      PRINT *,' '
      PRINT *,'OUTSIDE EDGE=',EDGE2
      ED2R = (Q + BGS)**2
      RW = (ED1R - ED2R) * ER
      PRINT *,' '
      PRINT *,'RESONANCE WIDTH (M)=',RW
C  *****************
C     CALCULATE LIBRATION PERIOD
C  *****************
C     CALCULATE SMALL DISPLACEMENT NEAR STABLE POINT
      DELSS = TPI/1000.
      DELBS = (EDGE2 - EDGE1)/1000.
C     NUMERICALLY DETERMINE SECOND PARTIALS
      CALL FDF(STBLE,SMS,FSTR,SDFDBS,SDFDSS)
      BGS = STBLE + DELBS
      CALL FDF(BGS,SMS,FSTR,DFDBS,DFDSS)
      DFDBS2 = (DFDBS - SDFDBS)/DELBS
      SMS = SMS + DELSS
      CALL FDF(STBLE,SMS,FSTR,DFDBS,DFDSS)
      DFDSS2 = (DFDSS - SDFDSS)/DELSS
C     DETERMINE FREQUENCY SQUARED
      FREQSQ = ABS(DFDBS2 * DFDSS2)
C     CALCULATE PERIOD
      PER = NO/SQRT(FREQSQ)
      PRINT *,' '
      PRINT *,'DELBS=',DELBS,' DELSS=',DELSS,' DFDBS2=',DFDBS2,
     2 ' DFDSS2=',DFDSS2
      PRINT *,' '
      PRINT *,'LIBRATION PERIOD (DAYS)=',PER
      RETURN
      END
```

```
      SUBROUTINE CONTUR(X,Y,DEL,XNEW,YNEW,IOK,ICAS,FF)
C  ****************
C     ROUTINE TO DETERMINE NEW POINT ON CONTOUR GIVEN OLD
C     ON INPUT X,Y IS PT ON CONTOUR
C     FUNCTION VALUE ON CONTOUR FF IS DEFINED ON FIRST CALL
C     ICAS IS ZERO ON FIRST CALL, USED TO FOLLOW CONTOUR IN SAME
C     SENSE THROUGHOUT AS SLOPE CASE SWITCHES
C     EACH CALL UPDATES TO NEW POINT ON CONTOUR, ONE PER CALL
C  ****************
C     TOLERANCES.....
C     TEN FIGURES IN X OR Y .......
      TOLXY = 1E-10
C     ....... OR FOURTEEN IN THE FUNCTION
      TOLF = 1E-14
C     ASSUME SUCCESS
      IOK = 0
C     FUNCTION VALUE AND PARTIALS
      CALL FDF(X,Y,F,DFDX,DFDY)
C     STORE FUNCTION VALUE ON FIRST CALL
      IF(ICAS .EQ. 0) FF = F
C
C     BRANCH ON CASE
C
      IF(ABS(DFDY/X) .LT. ABS(DFDX)) GO TO 500
C  ****************
C     SHALLOW SLOPE, FIX X DETERMINE Y
C  ****************
C     FIRST CALL ?
      IF( ICAS .EQ. 0) ICAS = -1
C     HAS CASE SWITCHED?
      IF(ICAS .EQ. -1) GO TO 50
C     CASE SWITCHED - FIX DEL TO CONTINUE TO FOLLOW IN SAME DIRECTION
      DEL = DEL * SIGN(1.0,DFDX) * SIGN(1.0,DFDY)*(-1.0)
      ICAS = -1
   50 CONTINUE
      DELX = DEL * ABS(X)
      XNEW = X + DELX
      YNEW = Y
      DY = -DFDX*DELX/DFDY
C     NEWTON LOOP
      DO 100 I = 1,30
      YNEW = YNEW + DY
C     RESIDUAL AND SLOPE
      CALL FDF(XNEW,YNEW,FP,DFPDX,DFPDY)
      RES = FP - FF
      DY = -RES/DFPDY
C     CONVERGED?
C     ABSOLUTE AND RELATIVE FUNCTION ERROR
      ERRF1 = ABS(RES)
      ERRF2 = 1.0
      IF(FF .NE. 0.0) ERRF2 = ABS(ERRF1/FF)
      ERRF = AMIN1(ERRF1,ERRF2)
      IF(ERRF .LT. TOLF) GO TO 1000
```

```
C     ABSOLUTE AND RELATIVE POSITION ERROR
      ERRXY1 = ABS(DY)
      ERRXY2 = 1.0
      IF(YNEW .NE. 0.0) ERRXY2 = ABS(DY/YNEW)
      ERRXY = AMIN1(ERRXY1,ERRXY2)
      IF(ERRXY .LT. TOLXY) GO TO 1000
  100 CONTINUE
C     FAILURE
      IOK = -1
      RETURN
C  *****************
C     STEEP SLOPE CASE, FIX Y, DETERMINE X
C  *****************
C     FIRST CALL ?
  500 IF(ICAS .EQ. 0) ICAS = 1
C     SWITCHED?
      IF(ICAS .EQ. 1) GO TO 501
C     IT SWITCHED
      DEL = DEL * SIGN(1.0,DFDX) * SIGN(1.0,DFDY)*(-1.0)
      ICAS = 1
  501 CONTINUE
      XNEW = X
      YNEW = Y + DEL
      DX = -DFDY*DEL/DFDX
      DO 600 I = 1,30
      XNEW = XNEW + DX
      CALL FDF(XNEW,YNEW,FP,DFPDX,DFPDY)
      RES = FP-FF
      DX = -RES/DFPDX
C     CONVERGED ?
C     ABSOLUTE AND RELATIVE FUNCTION ERROR
      ERRF1 = ABS(RES)
      ERRF2 = 1.0
      IF(FF .NE. 0.0) ERRF2 = ABS(ERRF1/FF)
      ERRF = AMIN1(ERRF1,ERRF2)
      IF(ERRF .LT. TOLF) GO TO 1000
C     ABSOLUTE AND RELATIVE POSITION ERROR
      ERRXY1 = ABS(DX)
      ERRXY2 = 1.0
      IF(XNEW .NE. 0.0) ERRXY2 = ABS(DX/XNEW)
      ERRXY = AMIN1(ERRXY1,ERRXY2)
      IF(ERRXY .LT. TOLXY) GO TO 1000
  600 CONTINUE
C     FAILURE
      IOK = -1
      RETURN
C     SUCCESS
 1000 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FDF(BGS,SMS,FSTR,DFDBS,DFDSS)
C  *****************
C     ROUTINE TO EVALUATE HAMILTONIAN AND FIRST PARTIALS
C     BGS AND SMS ARE INPUT CONJUGATE ACTION-ANGLE VARIABLES
C  *****************
      REAL J2,J2SQ,J4,J4OVR,J22,NO
      COMMON /SAOIII/ J2,J2SQ,J4,J4OVR,J22,NO,PHI22
      COMMON /PARAM/ Q,R,RADIAN,TPI
      QPS = Q+BGS
C     COMPUTE COEFFICIENTS FOR EQUATIONS
      A = (Q+BGS)/(Q+2*BGS)
      B = (2*BGS+R)/(Q+2*BGS)
C  *****************
C     FIRST RESONANCE TERM HAMILTONIAN
C  *****************
      FSTR = 1/(2*QPS**2)+J2*A**3/(2*QPS**6)*(3/2*B**2-1/2)
     2 +J2SQ*A**5/(4*QPS**10)*
     3 (15/32+27/32*J4OVR-(27/16+135/16*J4OVR)*B**2+(15/32+315/32*J4OVR)
     4 *B**4+3/8*A*(1-6*B**2+9*B**4)-A**2*(15/32+45/32*J4OVR
     5 -(15/16+225/16*J4OVR)*B**2-(105/32-525/32*J4OVR)*B**4))
     6 -3*J22*SQRT(-BGS)/(4*SQRT(2.0)*QPS**6.5)*(1+B)**2*
     7 COS(SMS-2*PHI22)+2*NO*BGS
C  *****************
C     PARTIAL OF F* WITH RESPECT TO BIG S
C  *****************
      DFDBS = -1/QPS**3-3/2*J2*A**3/QPS**7*((3*Q+4*BGS)/(Q+2*BGS)*
     2 (3/2*B**2-1/2)-A*B*(2*Q-2*R)/(Q+2*BGS))-5/4*J2SQ*A**5/
     3 QPS**11*((3*Q+4*BGS)/(Q+2*BGS)*
     4 (15/32+27/32*J4OVR-(27/16+135/16*J4OVR)*B**2+(15/32+315/32*J4OVR)
     5 *B**4+3/8*A*(1-6*B**2+9*B**4)-A**2*(15/32+45/32*J4OVR
     6 -(15/16+225/16*J4OVR)*B**2-(105/32-525/32*J4OVR)*B**4))
     7 -A/5*((2*Q-2*R)/(Q+2*BGS)*(-(27/8+135/8*J4OVR)*B+(15/8+315/8*
     8 J4OVR)*B**3+3/8*A*(36*B**3-12*B)+A**2*((15/8+225/8*J4OVR)*B
     9 +(105/8-525/8*J4OVR)*B**3))-Q/(Q+2*BGS)*(3/8*(1-6*B**2+9*B**4)
     A -2*A*(15/32+45/32*J4OVR-(15/16+225/16*J4OVR)*B**2
     B -(105/32-525/32*J4OVR)*B**4))))
     C -3*J22/(8*SQRT(2*(-BGS))*QPS**7.5)*((12*BGS-Q)*(1+B)**2
     D -(2*Q-2*R)/(Q+2*BGS)*4*BGS*A*(1+B))*COS(SMS-2*PHI22)+2*NO
C  *****************
C     PARTIAL OF F* WITH RESPECT TO SMALL S
C  *****************
      DFDSS = 3*J22*SQRT(-BGS)/(4*SQRT(2.0)*QPS**6.5)*
     2 (1+B)**2*SIN(SMS-2*PHI22)
      RETURN
      END
```

Vita

Robert Ian Boren was born on 8 October 1949 in Fergus Falls, Minnesota. He graduated from high school there in 1967 and attended the United States Air Force Academy, Colorado, from which he received the degree of Bachelor of Science in Physics and a regular commission in the United States Air Force in June 1971. After graduation, he served as an orbital analyst at the Space Defense Center outside of Colorado Springs, Colorado. From June 1974 to June 1975 he was a Sensor Space Systems Controller at a Spacetrack radar site in Turkey. After returning from overseas, he spent 4½ years with the 4000th Aerospace Applications Group, Omaha, Nebraska where he became Chief of the Astrodynamic Software Section. From there, he went to Los Angeles AFS, California where he served as a Strategic Air Command liaison to the NAVSTAR Global Positioning System Joint Program Office until entering the School of Engineering, Air Force Institute of Technology, in June 1981.

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An investigation of the spin-orbit resonance effects for a semi-synchronous near polar orbit was undertaken in order to determine whether consideration of all resonance terms associated with a particular commensurability ratio would result in the existence of multiple equilibrium points for the placement of satellites utilizing this type of orbit. The Hamiltonian of the geopotential in Delaunay elements using first and second

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order zonal harmonic terms (J_2, J_2^2, and J_4) was first transformed to a set of modified variables. The effects of the resonant disturbing function were then developed and the resultant Hamiltonian, valid for near polar inclinations and small eccentricities, was reduced to a single degree of freedom through a second transformation. Phase portraits of the system Hamiltonian were then generated and an analysis of the resonance structure and librational periods performed.

A single equilibrium point for each resonance term was discovered. Because of the physical meaning of the critical argument, s, the stable equilibrium points of both the first and second resonance terms appear to be candidates for deployment of future satellite systems. In addition, the semi-analytic technique of phase portraits was shown to be a feasible approach to the investigation of resonance effects.

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